Eliashberg Function $\alpha^2(E)F(E)$ and the Strong-Coupling Behavior of a Disordered Superconductor

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The electron-phonon interaction in a disordered superconductor is calculated. One finds additional electron-phonon processes which do not conserve the lattice momentum and a reduction of the normal and the umklapp processes. The additional electron-phonon processes contribute particularly to processes with low energy transfer, so that the Eliashberg function depends linearly on the energy in the low-energy region. This is in agreement with recent tunneling experiments. The linear part of the Eliashberg function causes the strong-coupling behavior of disordered superconductors. Therefore, the anomalous properties of a disordered superconductor can be explained by the change in the electron-phonon interaction.

I. INTRODUCTION

Weak-coupling superconductors, condensed onto a substrate at helium temperature, show an increased transition temperature[1-3] and energy gap.[4-6] Also, these quenched superconductors have an enlarged ratio $2\Delta_0/kT_c$, which means that they behave like strong-coupling superconductors. Recently Zavaritskii,[7] Knorr and Barth,[8] and Wühl[9] measured the Eliashberg function $\alpha^2(E)F(E) = G(E)$ for disordered superconductors. They found that $G(E)$ varies, particularly at low energies, where it depends linearly on the energy in disordered superconductors. These authors interpreted the variation of $G(E)$ by the assumption that the disordered superconductor has additional low-energy phonons. The same supposition was made by Garland, Bennemann, and Müller,[10] who attempted to explain the change of the transition temperature in disordered superconductors. Ewert[11] refuted this assumption by measuring the specific heat of disordered and annealed lead and indium films. He found that there was no change in the low-temperature specific heat, and concluded from his measurements that there are no additional low-frequency phonons in disordered superconductors. Markowitz and Kadanoff[12] calculated the change of T_c as a function of the mean free path l. They found a smearing of the anisotropy (which we do not consider here) and, in addition, a small change of T_c proportional to $1/l$, which they included in the valence effect. Ginsberg[13] showed that the attenuation constant of ultrasonic waves depends on the mean free path of the electrons, and he concluded that the electron-phonon interaction is increased for a short mean free path. The author[6,14] showed recently that the electron-phonon interaction in a disordered superconductor is enhanced, and found good agreement with the experimental change in the energy gap. Karaivanov[15] calculated a change in the electron-phonon interaction in alloys which resulted in an increase of the transition temperature. Maksimov[16] considered the influence of crystal defects on the superconducting transition temperature by means of the correlation scattering function.

In this paper we intend to derive the electron-phonon interaction and the Eliashberg function $G(E)$ in the rigid-ion approximation for a simple model of a disordered superconductor. We use plane waves for the electron states. Our aim will be to understand the experimental change of the function $G(E)$ and the fact that disordered superconductors are strong coupling.

II. MODEL OF A DISORDERED METAL

The structure of a quenched film is very complex. It is somewhere between the long-range order of the periodic crystal and the short-range order of a liquid. Therefore it appears to be difficult to calculate the properties of a realistic film. But we want to make a first step in this direction by considering a very simple model of the disordered film. We only require that our model of the disordered metal has a residual resistance at low temperature. For developing this model we begin with a periodic crystal where the atoms are located at the positions $\vec{r}_j^{\,0}$. Now we displace the atom j at the position $\vec{r}_j^{\,0}$ by a small displacement $\delta\vec{r}_j$. The final position of the atom j will be $\vec{r}_j^{\,0} + \delta\vec{r}_j$. The displacements $\delta\vec{r}_j$ shall be completely statistical and given by a Gaussian distribution

$$\exp[-(\delta\vec{r}_j)^2/2\delta^2]. \tag{2.1}$$

In a real disordered metal the displacements of the atoms are preserved by internal forces which keep the atoms in the displaced position. However,

in our simple model the displacements would be unstable without external forces. Therefore we have to introduce external forces $\vec{X}_j$ which act on the atoms j. The external forces may be arranged in such a way that the new positions $\vec{r}_j^0 + \delta\vec{r}_j$ of the atoms are the new equilibrium positions.

III. PHONON SPECTRUM

Now we want to test whether the spectrum of the phonons is changed in our simple model of a disordered metal. First, for the sake of simplicity, we consider a linear chain. Let j be the number of an atom, and $\vec{r}_j^0$ its equilibrium position in the unperturbed periodic chain: $\vec{r}_j^0 = bj$, where b is the lattice parameter. For the unperturbed chain we have the classical equation of motion for the elongation s_j:

$$M\ddot{s}_j = D(s_{j-1} + s_{j+1} - 2s_j)\,, \tag{3.1}$$

where M = ion mass and D = interatomic force constant. Using the ansatz $s_j \propto e^{i(jbq-\omega t)}$, the dispersion relation between ω and q, and hence the phonon spectrum, may be found. Now we consider a disordered linear chain. We introduce displacements δr_j from the periodic position by imposing forces X_j on the atom j. The new equation of motion is

$$M\ddot{s}_j = X_j + D(s_{j-1} + s_{j+1} - 2s_j) + D(\delta r_{j-1} + \delta r_{j+1} - 2\delta r_j). \tag{3.2}$$

This equation still holds for vanishing lattice oscillations $\ddot{s}_j = s_j = s_{j+1} = s_{j-1} = 0$ and determines the forces X_j:

$$0 = X_j + D(\delta r_{j-1} + \delta r_{j+1} - 2\delta r_j). \tag{3.3}$$

If we subtract this from Eq. (3.2) we find the same equation of motion as for the unperturbed chain. It can be solved by the same ansatz as before. These considerations can be generalized for the three-dimensional crystal. We find that the phonon spectrum in this simple model of a disordered metal remains unchanged and that the phase of the elongation $\vec{s}_j$ is determined by the unshifted position of the atoms $\vec{r}_j^0$:

$$\vec{s}_j \propto e^{i(\vec{q}\cdot\vec{r}_j^0 - \omega t)}\,. \tag{3.4}$$

An unchanged phonon spectrum is surely a simplification for a real disordered metal. However, Ewert[11] concluded from his measurements of the specific heat that, at least for the low energies, the phonon spectrum is unaltered. This appeared to be in contradiction with recent tunneling experiments where the change of the Eliashberg function $G(E)$ was measured. We will try to resolve this contradiction in Sec. V.

IV. ELECTRON-PHONON INTERACTION

A. Heuristic Description

In this section we wish to give a heuristic description of the electron-phonon processes in a disordered metal, whereas in Sec. IV B we give a quantitative calculation. In Fig. 1(a) we have an arrangement of atoms in the form of a periodic crystal. Let an electron wave with momentum $\vec{k}$ enter from the left. This electron wave is scattered by each atom which is producing an elementary wave. When one superimposes the amplitudes of the elementary waves one finds that they cancel in all directions, except in the original direction $\vec{k}$ and the directions $\vec{k} + \vec{g}_n$ ($\vec{g}_n$ = reciprocal-lattice vector). In Fig. 1(b) we have the same entering electron wave, but now the atoms of the crystal are not at rest but oscillate with the frequency ω_{ph} because a sound wave with the frequency ω_{ph} and the momentum q propagates through the crystal. Therefore the elementary electron waves are modulated in phase. We will separate the modulated wave into one electron wave with an unshifted frequency and two side lines whose frequencies are shifted by $\pm\omega_{ph}$. These shifted frequencies correspond to the absorption and emission of a phonon ω_{ph}. The emerging unshifted electron wave has the same direction as in Fig. 1(a), whereas the shifted waves have also a changed direction due to the space-dependent phase of the oscillation. Only the sum (or difference) of the electron and phonon momenta is equal to the original $\vec{k}$ (normal processes with momentum conservation) or $\vec{k} + \vec{g}_n$ (umklapp pro-

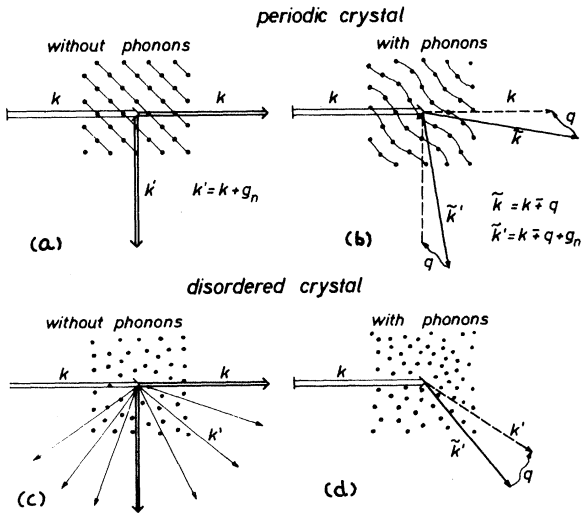

FIG. 1. Scattering of a plane electron wave in a periodic and a disordered crystal demonstrating the electron-phonon interaction.

cesses). This is the already known situation for a periodic crystal. In Fig. 1(c) we consider a disordered crystal where the arrangement of the atoms is rather arbitrary. Now the elementary waves do not cancel in any direction. The electron wave is scattered into each direction $\vec{k}'$ and the umklapp processes are reduced. In Fig. 1(d) we again have a sound wave which is propagating in the metal. (This is difficult to illustrate.) We obtain, as in Fig. 1(b), the normal electron-phonon processes. In addition, we find the modulated electron waves in every direction $\vec{k}'$. Again we can separate this modulated wave. It is important that there is no conservation of momentum between the electron and the phonon. In Fig. 1(d) we have $\vec{k}'+\vec{q}=\vec{k}'\neq\vec{k}$. We call these processes pseudo-umklapp-processes.

B. Calculation

A single atom of the disordered metal is given the potential

$$v(\vec{r})=\sum_{\vec{p}} v_{\vec{p}}\, e^{i\vec{p}\cdot\vec{r}}\,. \tag{4.1}$$

We assume that the potential is rigid and independent of the arrangement of the atoms. The positions of the atoms are $\vec{r}_j=\vec{r}_j^{\,0}+\delta\vec{r}_j$. We calculate the matrix elements of the Hamiltonian with the functions of the free electrons. We obtain for $V_{\vec{k}'\vec{k}}=V_{\vec{k}'-\vec{k}}$

$$V_{\vec{k}'\vec{k}}=v_{\vec{k}-\vec{k}}\ \sum_j e^{i(\vec{k}-\vec{k}')\cdot\vec{r}_j}\,. \tag{4.2}$$

We calculate the square of the matrix element and take the average over the statistical distributions with Eq. (2.1):

$$|V_{\vec{k}'\vec{k}}|^2=|v_{\vec{k}'-\vec{k}}|^2[L^2e^{-(\vec{k}-\vec{k}')^2\delta^2}\delta_{\vec{k}-\vec{k}',\vec{g}_n}+L(1-e^{-(\vec{k}-\vec{k}')^2\delta^2})], \tag{4.3}$$

where $\vec{g}_n$ = reciprocal-lattice vector.

The electron-phonon matrix element is calculated by introducing the lattice oscillation with the elongation s_j according to (3.4):

$$\vec{s}_j=\sum_{\vec{q}\lambda}(a_{\vec{q}\lambda}^{\dagger}+a_{-\vec{q}\lambda})\,\vec{e}_{\vec{q}\lambda}\,e^{-i\vec{q}\cdot\vec{r}_j^{\,0}}\,, \tag{4.4}$$

where $a_{\vec{q}\lambda}^{\dagger}$ and $a_{-\vec{q}\lambda}$ are the creation and annihilation operators of the phonons. $\vec{e}_{\vec{q}\lambda}$ is the polarization vector of the phonon with the wave vector $\vec{q}$ and the polarization λ and has the length

$$|e_{\vec{q}\lambda}|^2=\hbar/2LM\omega_{\vec{q}\lambda}\,, \tag{4.5}$$

where M = ion mass, L = number of atoms per unit volume, and $\omega_{\vec{q}\lambda}$ = frequency of the phonon $\vec{q}\lambda$. We obtain for the electron-phonon matrix element

$$g_{\vec{k}'\vec{k}\,\vec{q}\lambda}=i\,v_{\vec{k}'-\vec{k}}\,\vec{e}_{\vec{q}\lambda}\cdot(\vec{k}-\vec{k}')\sum_j\exp[i(\vec{k}-\vec{k}')\cdot\vec{r}_j-i\vec{q}\cdot\vec{r}_j^{\,0}] \tag{4.6}$$

and find for the statistical average of $|g_{\vec{k}'\vec{k}\vec{q}\lambda}|^2$

$$|g_{\vec{k}'\vec{k}\vec{q}\lambda}|^2=|v_{\vec{k}'-\vec{k}}|^2|\vec{e}_{\vec{q}\lambda}|^2|\vec{k}-\vec{k}'|^2\cos^2(\vec{e}_{\vec{q}\lambda},\vec{k}-\vec{k}')\times\{L^2e^{-(\vec{k}-\vec{k}')^2\delta^2}\delta_{\vec{k}-\vec{k}'-\vec{q},\vec{g}_n}+L[1-e^{-(\vec{k}-\vec{k}')^2\delta^2}]\}\,. \tag{4.7}$$

Therefore the electron-phonon processes which occur in the periodic crystal are reduced by the factor $e^{-(\vec{k}-\vec{k}')^2\delta^2}$ in the disordered state. This means that the normal processes, as well as the umklapp processes, are reduced. However we obtain additional electron-phonon processes which we called pseudo-umklapp processes and which do not conserve the lattice momentum.

The Hamiltonian operator has the general form

$$H=\sum_{\vec{k}}\frac{\hbar^2k^2}{2m}\,c_{\vec{k}}^{\dagger}c_{\vec{k}}+\sum_{\vec{k}\vec{k}'}V_{\vec{k}'\vec{k}}\,c_{\vec{k}'}^{\dagger}c_{\vec{k}}+\sum_{\vec{k}'\vec{k}\vec{q}\lambda}g_{\vec{k}'\vec{k}\vec{q}\lambda}\,c_{\vec{k}'}^{\dagger}c_{\vec{k}}(a_{\vec{q}\lambda}^{\dagger}+a_{-\vec{q}\lambda})\,. \tag{4.8}$$

The matrix elements $|V_{\vec{k}'\vec{k}}|^2$ and $|g_{\vec{k}'\vec{k}\vec{q}\lambda}|^2$ have two different contributions. One part conserves lattice momentum; there both $|V|^2$ and $|g|^2$ are proportional to L^2. The other part does not conserve the lattice momentum causing the scattering of the electrons and the pseudo-umklapp-processes. This part is proportional to L. The part of $|V_{\vec{k}'\vec{k}}|^2$ which is proportional to L^2 determines the shape of the Fermi surface. The second contribution of $|V_{\vec{k}'\vec{k}}|^2$ which is proportional to L describes the scattering of the electrons, their resistance, and finite lifetime. In the following calculation we neglect the off-diagonal elements and approximate the Fermi surface by a sphere and the eigenstates by plane waves. We find a simple relation between the electron-phonon matrix elements of the pseudo-umklapp-processes and the scattering matrix elements (i.e., the terms which are proportional to L):

$$|g_{\vec{k}'\vec{k}\vec{q}\lambda}|^2=|V_{\vec{k}'\vec{k}}|^2|\vec{k}-\vec{k}'|^2\,\frac{\hbar}{2LM\omega_{\vec{q}\lambda}}\cos^2(\vec{e}_{\vec{q}\lambda},\vec{k}-\vec{k}')\,. \tag{4.9}$$

V. ELIASHBERG FUNCTION

In Sec. IV we obtained a changed electron-phonon interaction in a disordered superconductor. Now it would be interesting to calculate the energy gap and the transition temperature of the disordered superconductor. The author[6] has given such a calculation neglecting the reduction of the normal and umklapp processes. This calculation required simplification in order to avoid solving the Eliashberg equation, which requires a computer calculation. The Eliashberg equation determines, in principle, both $2\Delta_0$ and T_c if $G(E)$ and the Coulomb pseudopotential $\mu^{\dagger}$ are given. $G(E)$ is (without a factor $\hbar/2\pi$) the probability of an electron of the energy E_1 to make a transition into any state of

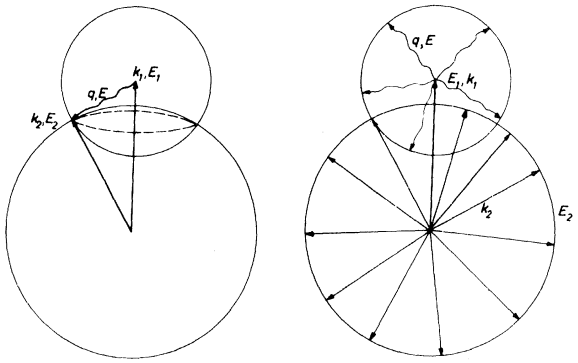

FIG. 2. Electron-phonon transitions for normal processes and pseudo-umklapp-processes.

the energy E_2 by emitting a phonon of the energy $E=\hbar\omega=E_1-E_2$ (Fig. 2). This transition probability is factorized into the product $\alpha^2(E)F(E)$, where $\alpha^2(E)$ is the averaged electron-phonon matrix element $|g_{\vec{k}\cdot\vec{k}\vec{q}\lambda}|^2$ and $F(E)$ is the density of phonon states. $G(E)$ has been determined recently by Zavaritskii, Knorr and Barth, and Wühl for disordered superconductors. They measured the derivative of the tunnel characteristic and refolded it [assuming that the Eliashberg equation is still correct for disordered superconductors if one uses the correct $G(E)$ and $\mu^\dagger$]. They found that $G(E)$ is changed in the disordered superconductors and that for small energies it is proportional to the energy. We will show that the change at low energies can be explained by the change of the electron-phonon interaction.

Again we use free-electron states as the eigenstates. At first we will discuss the energy dependence of $G(E)$ (see Fig. 2). It is proportional to the square of the matrix element $|g_{\vec{k}_1\vec{k}_2\vec{q}\lambda}|^2$ times the number of final states (which is not equal to the density of phonon states). For normal processes it is $|g_{\vec{k}_1\vec{k}_2\vec{q}\lambda}|^2\propto\omega^{-1}|k_1-k_2|^2\propto q^2/\omega\propto E$. The number of final states is restricted by the conservation of momentum and is proportional to $q\propto E$. Therefore we have for normal processes $G(E)\propto E^2$. For pseudo-umklapp-processes we do not have to conserve the momentum. The final phonon states lie on the surface of a sphere with the radius $q=\omega/c=E/\hbar c$ and their number is proportional to E^2. The number of final electron states is independent of the energy and equal to the density of electron states $N(E_F)$. The averaged matrix element $|g_{\vec{k}_1\vec{k}_2\vec{q}\lambda}|^2\propto\omega^{-1}|\vec{k}_1-\vec{k}_2|^2\propto 1/E$, because $|\vec{k}_1-\vec{k}_2|^2$ does not depend on the energy. Therefore we find in the disordered superconductor that $G(E)$ is proportional to E. A detailed calculation gives

$$G(E)=\sum_{\vec{k}_2}\sum_{\vec{q}}\sum_{\lambda}|g_{\vec{k}_2\vec{k}_1\vec{q}\lambda}|^2\,\delta(E-\hbar cq)\delta(E_1-E_2-E). \tag{5.1}$$

After transforming the sum over $\vec{k}_2$ into an integral over the Fermi surface and the sum over $\vec{q}$ into an integral over $4\pi q^2dq$, and introducing Eq. (4.9) for low energy or momentum, we find

$$G(E)=\frac{1}{64\pi^6}\sum_{\lambda}\int 4\pi q^2dq\,\frac{\hbar}{2LMc_\lambda q}\int\frac{dS}{\hbar v_F}\,|\vec{k}_1-\vec{k}_2|^2\times|V_{\vec{k}_1-\vec{k}_2}|^2\cos^2(\vec{e}_{\vec{q}\lambda},\vec{k}_1-\vec{k}_2)\,\delta(E-\hbar cq). \tag{5.2}$$

With $\langle\cos^2(\vec{e}_{\vec{q}\lambda},\ \vec{k}_1-\vec{k}_2)\rangle_{\rm av}=\frac{1}{3}$ we obtain

$$G(E)=\frac{mk_F}{24\pi^4LM\hbar^3}\left(\frac{1}{c_l^3}+\frac{2}{c_{\rm tr}^3}\right)\langle|\vec{k}_1-\vec{k}_2|^2|V_{\vec{k}_1-\vec{k}_2}|^2\rangle_{\rm av}E. \tag{5.3}$$

c_l and $c_{\rm tr}$ are the transverse and longitudinal sound velocities. The average of $|\vec{k}_1-\vec{k}_2|^2|V_{\vec{k}_1-\vec{k}_2}|^2$ is taken over the Fermi surface and can be expressed by the mean free path of the electrons,

$$\langle|\vec{k}_1-\vec{k}_2|^2|V_{\vec{k}_1-\vec{k}_2}|^2\rangle_{\rm av}=\frac{2\pi\hbar^4k_F^2}{m^2}\,\frac{1}{l}\,. \tag{5.4}$$

Finally we introduce the number of electrons per unit volume

$$n=(1/3\pi^2)k_F^3 \tag{5.5}$$

and obtain

$$G(E)=\frac{\hbar n}{4\pi LMm}\left(\frac{1}{c_l^3}+\frac{1}{c_{\rm tr}^3}\right)\frac{1}{l}\,E\,. \tag{5.6}$$

$G(E)$ indeed depends linearly on the energy, as was found in the tunneling experiments.

The energy dependence of $G(E)$ for large energy can be discussed only qualitatively, because its calculation is difficult even in a periodic crystal and depends very much on the detailed structure of the Fermi surface. The same difficulty occurs in the disordered superconductor. We obtain, however, from Eq. (4.7) that $G(E)$ is reduced for the normal processes as well as for the umklapp processes.

VI. COMPARISON WITH EXPERIMENT

We make a comparison between the calculated slope of the Eliashberg function $G(E)$ at low energies and the measured slope by Knorr and Barth for tin and lead. We use for tin

$$LM=7.3\times10^3\ \text{kg/m}^{-3},\quad n=1.5\times10^{29}\ \text{m}^{-3},$$
$$c_l=3.3\times10^3\ \text{m/sec},\quad c_{\rm tr}=1.7\times10^3\ \text{m/sec},$$
$$l_{\rm tr}=65\times10^{-10}\ \text{m}$$

and for lead

$$LM=11.5\times10^3\ \text{kg/m}^3,\quad n=1.3\times10^{29}\ \text{m}^{-3},$$
$$c_l=2.05\times10^3\ \text{m/sec},\quad c_{\rm tr}=7.1\times10^2\ \text{m/sec},$$
$$l_{\rm tr}=50\times10^{-10}\ \text{m}\,.$$

We obtain from these the following values: For tin, $G(E) = 4.2$ or 1 $(\mathrm{eV})^{-1}$ and for lead, $G(E) = 30$ or 10 $(\mathrm{eV})^{-1}$ for experimental and theoretical values, respectively. The theoretical slope is smaller by a factor of 3–4 than the experimental value. This may be caused by the free-electron approximation.

The reduction of the normal and umklapp processes which we found in the theory is in qualitative agreement with the experiment, where, except for the low-energy region, $G(E)$ is slightly reduced in the disordered states.

VII. STRONG-COUPLING EFFECT OF DISORDERED SUPERCONDUCTORS

With the exact knowledge of $G(E)$, one can calculate the energy gap Δ_0 and the transition temperature T_c by solving the Eliashberg equation. The author is not able to do such a computer calculation yet. However, we wish to remark that the unexpected property of strong coupling in a disordered superconductor may be understood qualitatively. As we saw, the function $G(E)$ for a disordered superconductor has a finite slope at low energies. The strong-coupling parameter $\lambda = 2\int G(E)\, dE/E$ weighs the energy dependence of $G(E)$ with the factor E^{-1} and favors the low-energy contribution. Therefore, it is the change of the electron-phonon interaction and its contribution to low-energy transitions that cause the strong-coupling behavior of disordered superconductors. Furthermore, the parameter determines the phonon renormalization of the electron mass. Therefore, we expect an increase of the effective electron mass and the density of electron states in a disordered metal.

VIII. CONCLUSION

The electron-phonon processes in a disordered superconductor have been discussed. We found additional electron-phonon processes which do not conserve the lattice momentum. These processes contribute to the Eliashberg function $G(E)$ at low energies and are responsible for the strong-coupling behavior of disordered superconductors. Therefore it is not necessary to assume that the phonon spectrum in a disordered superconductor is changed to explain its anomalous properties. We used the free-electron eigenfunctions for the whole calculation. It would be very useful to extend the calculation by using the correct eigenfunction of the disordered superconductor. Being an experimentalist the author would like to leave such a calculation to a theorist. The results also apply in many respects to superconducting alloys and amorphous superconductors.

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